

ODDERON IN QCD

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A review on how the Odderon idea does appear in QCD is given. In the last years it has been developed a non-perturbative QCD approach based on the stochastic vacuum model and a perturbative one based on resummation techniques in the small x QCD region. Last developments on the perturbative analysis are shown in some details, in particular in application to diffractive η_c production.

1 Introduction

It is remarkable that QCD predicts the existence of the Odderon. The idea of the Odderon¹, the partner of the Pomeron which is odd under parity P and charge conjugation C (like the photon), is related to the possibility that the real part of a scattering amplitude increases with energy as fast as the imaginary part. The scattering amplitude in the complex angular momentum plane possesses a rightmost singularity (pole) near $j = 1$. In the even (under crossing) amplitude such a singularity is associated to the Pomeron and gives a mostly imaginary contribution, while in the odd case one has a mostly real contribution which is associated to the Odderon. The position of the singularity is also called intercept and it is related to the asymptotic behaviour of the cross section.

The very successful theory for the strong interaction, QCD, predicts the Pomeron existence, in the most simple version as a two gluon exchange in colour singlet state, thanks to the vector nature of the gluon.

The fact that the internal gauge symmetry group of QCD has rank greater than one permits to construct from three gluons a C -odd state which can be associated to the Odderon.

One can see this fact considering the $SU(3)_C$ gauge group associated to the gluon field $A_\mu = \sum_a A_\mu^a t_a$. Since under charge conjugation one has $A_\mu \rightarrow -A_\mu^T$, the two possible independent invariants, constructed by three gluon fields, are $Tr([A_1, A_2]A_3)$ and $Tr(\{A_1, A_2\}A_3)$, which are respectively even and odd under charge conjugation. Therefore the Odderon will be related to the composite operator $O_{\alpha\beta\gamma} = d_{abc}A_\alpha^a A_\beta^b A_\gamma^c$.

What is challenging the physics community is that experimentally there is till now no clear evidence of the Odderon. Among the ways to look at his

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presence there is the comparison of the total and/or elastic cross sections for direct and cross-symmetric scattering processes, like for example in the case of pp and $p\bar{p}$ scattering. It was infact in this context that the Odderon was originally introduced.

Another class of scattering processes, where the Odderon contributes, is when one or two of the incoming scattering particles, of definite C-parity, goes into a state of opposite C-parity under scattering. One requires a rapidity gap which allows to separate the outgoing scattering states. A typical example is given by the reaction

$$\gamma(\gamma^*) + p \rightarrow PS(T) + p(X_p),$$

where a photon scatters a proton and a pseudoscalar or a tensor meson is produced in the photon fragmentation region, well separated in rapidity from the proton or its debris (X_p). This process has started to be analyzed at HERA ². The $\gamma\gamma$ scattering process ³ could be also interesting, even if the cross section involved are much smaller .

In this direction perturbative analysis have been performed in the study of η_c production in DIS with an Odderon made by three simply uncorrelated gluons ^{4,5} and later by considering the resummed QCD interaction in LLA ⁶. Some details of the last approach are given in the section 4.

Non perturbative studies have been carried on for the production of light mesons (π^0 , f_2) ⁷. This general non perturbative approach will be sketched in section 3. The π^0 production process has been very recently analyzed at HERA by the H1 collaboration, the Odderon has not been seen and it has been put an upperbound on the cross section ten times smaller than the predicted cross section ⁸, setting a new challenge for the theory understanding.

Another interesting proposal, based on a more phenomenological approach, has been the study of charge asymmetry in charm states due to Pomeron-Odderon interference ⁹.

2 Perturbative QCD Odderon in LLA

A scattering process dominated by the Odderon exchange can be described in the high energy limit, in the context of k_T factorization, by an amplitude

$$A(s, t) = \frac{s}{32} \frac{1}{16} \frac{N_c^2 - 4}{N_c} \frac{N_c^2 - 1}{3!} \frac{1}{(2\pi)^8} \langle \Phi_\gamma^i | G_3 | \Phi_p \rangle. \quad (1)$$

At lowest order, provided the strong coupling α_s is small, one has a simple three uncorrelated gluon exchange, i.e. the Green function G_3 , which is convoluted with the impact factors, is constructed, simply with 3 gluon propaga-

tors. Therefore, in momentum representation $G_3^{(LO)} = \delta^{(2)}(\mathbf{k}_1 - \mathbf{k}'_1) \delta^{(2)}(\mathbf{k}_2 - \mathbf{k}'_2) 1/k_1^2 k_2^2 k_3^2$.

In the high energy limit, when all other physical invariants are much smaller, a LLA resummation of the contributions of the order $(\alpha_s \ln s)^n$, which is not small, can be performed and one obtains, through G_3 , an effective evolution in rapidity. The same resummation for the two gluon exchange has lead to the BFKL¹⁰ equation where it appears the kernel of the integral equation for the 2-gluon Green function that, in the colour singlet state, describes the perturbative QCD Pomeron in LLA. The same equation in the colour octet state has a simple eigenstate which corresponds to the reggeized gluon, which is in general at high energies a composed object. This fact is seen as a self consistency requirement and it is called bootstrap. In NLA¹¹, where one is resumming also the contribution of order $\alpha_s^n (\ln s)^{n-1}$, all the same concepts, reggeization included¹², apply.

The general kernel for the n -gluon integral equation for the Green function in LLA is given by the BKP equation¹³. In the large N_c limit and for finite N_c when $n = 3$, it possesses remarkable symmetry properties: discrete cyclic symmetry, holomorphic separability, conformal invariance, integrability, duality¹⁴ and also a relation between solutions with different n exists¹⁵, which is a direct consequence of the gluon reggeization.

The Odderon states in LLA must be symmetric eigenstates of the operator $K_3 = 1/2(K_{12} + K_{23} + K_{31})$ constructed with the BFKL kernel K_{ij} for two reggeized gluons in a singlet state. Using the conformal invariance and integrability properties a set of eigenstates has been found¹⁶, which have a maximal intercept below one.

Using the gluon reggeization property (bootstrap) a new set of solutions was later found¹⁷, characterized by intercept up to one, therefore dominant at high energies. Moreover for the particular impact factor which couples a photon and an η_c to the Odderon the LLA calculation has shown that this second set of solution is relevant while the previous one decouples. We present here these Odderon states, since they will be used in section 4. In momentum representation they are given by $E_3^{(\nu,n)}$ such that

$$k_1^2 k_2^2 k_3^2 E_3^{(\nu,n)}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = c(\nu, n) \sum_{(123)} (\mathbf{k}_1 + \mathbf{k}_2)^2 k_3^2 E^{(\nu,n)}(\mathbf{k}_1 + \mathbf{k}_2, \mathbf{k}_3), \quad (2)$$

where $c(n, \nu)$ is a normalization factor, E is a BFKL pomeron eigenstate and the conformal spin n is odd. The full Green function is constructed summing over all such states but in the high energy limit the asymptotic behaviour can be studied for conformal spin $n = \pm 1$ and performing the saddle point integration around $\nu = 0$.

3 Non perturbative QCD Odderon

A very briefly sketch on the non perturbative QCD framework used for Odderon studies ⁷ is given in the following.

A first ingredient is the choice of the eikonal semiclassical approximation¹⁸ for high energy scattering of quarks, while, at first, a full quantum colour field behaviour is considered. In particular each quark which scatters on a colour field picks up a non abelian eikonal phase $V = P \exp [-ig \int_{\Gamma} dz^{\mu} \mathbf{A}_{\mu}(z)]$.

The functional integral on the physical gluon field is estimated using the stochastic vacuum model (SVM) ¹⁹, i.e. the calculation of any correlation functions of gluon field strength is associated to a gaussian stochastic process with finite correlation length and, therefore, expanded as $\langle F \cdot F \cdots F \rangle = \sum \prod \langle F \cdot F \rangle$. After some other assumptions and relating the basic two point function $\langle 0|F \cdot F|0 \rangle$ to the gluon vacuum condensate, a dipole - dipole or dipole-tripole (as Wegner-Wilson loops) scattering amplitude at fixed transverse size can be computed expanding the ordered exponential. Mesons (barions) are described in term of dipoles (tripoles) and transverse wave functions ²⁰.

When expanding the exponentials in the eikonal phases, terms of the kind $\langle Tr(F \cdot F) Tr(F \cdot F) \rangle$ give imaginary contribution and are associated to the Pomeron. Instead the real Odderon contribution is given by subsequent terms of the kind $\langle Tr(F \cdot F \cdot F) Tr(F \cdot F \cdot F) \rangle$, in particular by the piece with the $d_{abc}d_{abc}$ colour structure.

In this approach the energy dependence can be introduced in a phenomenological way. In general a diquark structure of the hadrons is preferred. Regarding Odderon driven processes, in particular the production of light mesons in DIS⁷ has been studied (π^0 , f_2 with N^* resonances production). Predictions at HERA energies are $\sigma_{\gamma p \rightarrow \pi^0 N}^O \approx 400$ nb and $\sigma_{\gamma p \rightarrow f_2 N}^O \approx 21$ nb.

The first process has been recently analyzed at HERA by the H1 collaboration and there is now an upperbound on the cross section of around 39 nb⁸. It is important to understand such a big discrepancy. One possible source of error comes from the parameter fixing in SVM, but the most serious one seems to be the badly estimated $\gamma O \pi^0$ vertex. It seems therefore that the f_2 production process would be based on more solid estimates of the coupling.

4 Diffractive η_c photo and electro-production

In order to apply, to some extent, perturbative QCD to the calculations, one can look at processes where heavy quarks are involved. Diffractive η_c production in DIS has been studied at lowest order ^{4,5}. The calculations give $\sigma \approx 11$ pb at $Q^2 = 0$ and 0.1 pb at $Q^2 = 25$ GeV², with no energy dependence.

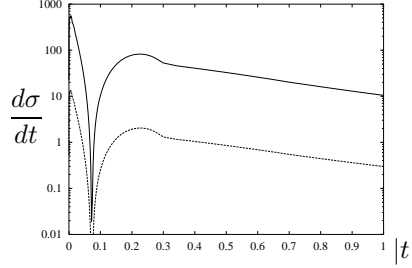


Figure 1. The differential cross sections (in pb / GeV²). The upper curve refers to $Q^2 = 0$.

This process has been recently reanalyzed⁶ in LLA. The amplitude (1) has been calculated in the saddle point approximation using the Green function G_3 , constructed with the non forward Odderon states in (2). One has

$$G_3(y) = \sum_{\text{odd } n} \int_{-\infty}^{+\infty} d\nu e^{y \chi(\nu, n)} N(\nu, n) E_3^{(\nu, n)} E_3^{(\nu, n)*}, \quad (3)$$

where $N(\nu, n)$ fix the representation dependence of the measure⁶. In order to compare the effect of LLA QCD resummation to the lowest order calculations, the same impact factors for the $\gamma O\eta_c$ and for pOp vertices have been used. The $\gamma O\eta_c$ impact factor has been computed perturbatively⁴; it has an interesting symmetry which allows a partial analytical computation^{17,6} of its scalar product with the Odderon eigenstates in (2). For the proton side the ansatz previously made⁴ has been used.

Due to the structure of the Odderon states, which manifests as a strong correlation between the constituent reggeized gluons, the Odderon coupling to the impact factor has the dominant real part which changes sign for a value of the momentum transfer squared t . The computation of the differential cross section leads therefore to the result presented in Fig. 1, where a dip in the small t region is present. Due to the cut nature of these Odderon singularities the cross section is slightly suppressed (as $1/\ln s$) with energy.

The total cross section, which results from the LLA Odderon states contribution, has been found to be $\sigma \approx 50$ pb at $Q^2 = 0$ and 1.3 pb at $Q^2 = 25$ GeV², an order of magnitude larger than in the simple three gluon exchange case. Quantitative and qualitative remarkable differences are introduced by the gluon interaction but the cross sections are small to be measured at HERA.

The Odderon still represents a challenge for theory and experiments.

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References

1. L. Lukaszuk and B. Nicolescu, Lett. Nuovo Cim. **8** (1973) 405.
2. A. Schafer, L. Mankiewicz and O. Nachtmann, UFTP-291-1992 *In *Hamburg 1991, Proceedings, Physics at HERA, vol. 1* 243-251 and Frankfurt Univ. - UFTP 92-291 (92,rec.Mar.) 8 p.*
3. I. F. Ginzburg and D. Y. Ivanov, collisions,” Nucl. Phys. B **388**, 376 (1992).
4. J. Czyzewski, J. Kwiecinski, L. Motyka and M. Sadzikowski, Phys. Lett. **B398** (1997) 400; erratum Phys. Lett **B411** (1997) 402.
5. R. Engel, D. Y. Ivanov, R. Kirschner and L. Szymanowski, Eur. Phys. J. **C4** (1998) 93.
6. J. Bartels, M. A. Braun, D. Colferai and G. P. Vacca, Eur. Phys. J. C **20**, 323 (2001).
7. M. Rueter, H. G. Dosch and O. Nachtmann, Phys. Rev. D **59** (1999) 014018. E. R. Berger, A. Donnachie, H. G. Dosch, W. Kilian, O. Nachtmann and M. Rueter, Eur. Phys. J. **C9** (1999) 491. E. R. Berger, A. Donnachie, H. G. Dosch and O. Nachtmann, Eur. Phys. J. C **14**, 673 (2000).
8. See the talk of T. Golling (H1) of this conference.
9. S. J. Brodsky, J. Rathsman and C. Merino, Phys. Lett. B **461**, 114 (1999).
10. E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. **JETP** **44** (1976) 443; **ibid.** **45** (1977) 199; Ya. Ya. Balitskii and L.N. Lipatov, Sov. J. Nucl. Phys. **28**, (1978) 822.
11. V. S. Fadin and L. N. Lipatov, Phys. Lett. B **429**, 127 (1998).
12. M. Braun and G. P. Vacca, Phys. Lett. B **454**, 319 (1999). M. Braun and G. P. Vacca, Phys. Lett. B **477**, 156 (2000). A. Papa, hep-ph/0007118.
13. J. Bartels, Nucl Phys. **B151**(1979) 293; Nucl Phys. **B175**(1980) 365; J. Kwieciński, M. Praszalowicz, Phys. Lett. **B94** (1980) 413.
14. L. N. Lipatov, Sov. Phys. JETP **63** (1986) 904; Phys. Lett. **B309** (1993) 394; JETP Lett. **59** (1994) 596; Sov. Phys. JETP Lett. **59** (1994) 571; Nucl. Phys. **B548** (1999) 328-362. L. D. Faddeev and G. P. Korchemsky, Phys. Lett. **B342** (1995) 311.
15. G. P. Vacca, Phys. Lett. B **489**, 337 (2000).
16. R. A. Janik and J. Wosiek, Phys. Rev. Lett. **82** (1999) 1092.
17. J. Bartels, L. N. Lipatov and G. P. Vacca, Phys. Lett. **B477** (2000) 178.
18. O. Nachtmann, Annals Phys. **209**, 436 (1991).
19. H. G. Dosch and Y. A. Simonov, Phys. Lett. B **205**, 339 (1988).
20. H. G. Dosch, E. Ferreira and A. Kramer, Phys. Rev. D **50**, 1992 (1994).